Guide to Imaginary Numbers.

How "real" are "imaginary" numbers?

The "real" numbers are well known. To the early Greeks these were the positive numbers 1,2,3… etc. Negative numbers did not exist (how can you have less than zero sheep, goats etc..). Real numbers can solve real algebraic equations like

$$x + 1 = 2$$

There is but one solution for this equation and that is $x = 1$. This equation is known as a first-degree equation. A second-degree equation like the one below has two solutions.

$$x^2 - 5x + 6 = 0$$

The solutions are $x = 2$ and $x = 3$. A third degree equation (x^3) has three solutions and a fourth degree equation has four. In fact at the turn of the 19-century, Karl Gauss showed that a $\boldsymbol{n}^{th}$ degree equation has $\boldsymbol{n}$ solutions. This is true for all equations. If so, what about :

$$x + 2 = 1$$

This first-degree equation has a solution : $x = -1$, a negative number, a number that the early Greeks would not like because to them it was not "real". For us, negative numbers are very "real" (ask anyone with a credit card!). Many algebraic equations can be solved with both positive and negative solutions (e.g. $x^2 + 4x - 5 = 0$; $x = 1$ and $x = -5$).

So far so good, now what about the very simple second-degree algebraic equation:

$$x^2 + 1 = 0$$

The only solution to this problem will set $x^2 = -1$ or $x = \sqrt{-1}$ (square root of –1). No positive or negative number will solve this equation and a solution is a must (in fact two), according to Gauss (is Gauss still happy?). For the solution of this equation we need a new "real" number system where $x^2 = -1$. This new "real" number system is of course "imaginary" and is called $i = \sqrt{-1}$ and the root of any negative number is $\sqrt{-n} = i\sqrt{n}$. Our new "real" imaginary number system has both positive and negative imaginary numbers i and $-i$. So back to $x^2+1 = 0$. There are two "real" solutions to this equation: $x = \sqrt{-1}$ and $-\sqrt{-1}$ and with these solutions Gauss is still smiling.