Guinier's Law

Beginning with the Amplitude equation

$$A(q) = \int \rho(r) e^{-iqr} dV$$

expand the exponital

$$A(q) = \int \rho(r) dV - i\int qr\rho(r) dV - \frac{1}{2!}\int (qr)^2 \rho(r) dV + ...$$

given $\int \rho(r) dV = \rho_0 V$ and $\int r\rho(r) dV = 0$ when the origin of *r* is at the center of mass (i.e. $\sum_{i=1}^{n} r_i = r_1 + r_2 - r_3 - r_4 ... = 0$ if r_i is centered at 0) of the particle then:

$$A(q) = \rho_0 V - 0 - \frac{1}{2}(qr)^2 \int \rho(r) dV + ...$$

$$A(q) = \rho_0 V \left[1 - \frac{1}{2}(qr)^2 ... \right]$$

The intensity is obtained by squaring the amplitude so :

$$I(q) = \langle A(q) \rangle^2 = \rho_0^2 V^2 \left[1 - \frac{1}{2} q^2 r^2 ... \right]^2$$

$$I(q) = \rho_0^2 V^2 \left[1 - q^2 r^2 + \frac{1}{4} q^4 r^4 ... \right]$$

For small q then $q^4r^4 <<< q^2r^2$ and the approximation

$$I(q) = \rho_0^2 V^2 \left[1 - q^2 r^2 \ldots\right]$$ is valid.

r is a vector that can be equated to the Cartesian coordinate system as

$$r^2 = \overline{x}^2 + \overline{y}^2 + \overline{z}^2 + 2\overline{xy} + 2\overline{xz} + 2\overline{yz}$$

and

$$q^2 r^2 = q_x^2 \overline{x}^2 + q_y^2 \overline{y}^2 + q_z^2 \overline{z}^2 + 2q_x^2 q_y^2 \overline{xy} + 2q_x^2 q_z^2 \overline{xz} + 2q_y^2 q_z^2 \overline{yz}$$

For an isotropic system $\overline{x}^2 = \overline{y}^2 = \overline{z}^2 = \frac{1}{3} R_g^2$, $\overline{xy} = \overline{xz} = \overline{yz} = 0$ and $q_x^2 = q_y^2 = q_z^2 = q^2$ therefore

$$I(q) \cong \rho_0^2 V^2 \left[1 - q^2 \frac{1}{3} R_g^2 \ldots\right]$$

or

$$I(q) \cong \rho_0^2 V^2 \exp\left(-\frac{1}{3} q^2 R_g^2\right)$$

Which is known as Guinier's Law (approximation).